

Fragmentation functions of $g \rightarrow \eta_c(^1S_0)$ and $g \rightarrow J/\psi(^3S_1)$ considering the role of heavy quarkonium spin

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The production of heavy quarkonia is a powerful tool to test our understanding of strong interaction dynamics. It is well-known that the dominant production mechanism for heavy quarkonia with large transverse momentum is fragmentation. In this work we, analytically, calculate the QCD leading order contribution to the process-independent fragmentation functions (FFs) for a gluon to split into the vector (J/ψ) and pseudoscalar (η_c) S -wave charmonium states. The analyses of this paper differ in which we present, for the first time, an analytical form of the $g \rightarrow J/\psi$ FF using a different approach (Suzuki's model) in comparison with other results presented in literatures, where the Braaten's scheme was used and the two-dimensional integrals were presented for the gluon FFs which must be evaluated numerically. The universal fragmentation probability for the $g \rightarrow J/\psi$ is about 10^{-6} which is in good consistency with the result obtained in the Braaten's model.

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I. INTRODUCTION

Heavy quarkonia, as the bound states of a heavy quark and antiquark, are the simplest particles when the strong interactions are concerned. Their production has a long history of theoretical calculations and experimental measurements [1], especially, their production at high-energy colliders has been the subject of considerable interest during the past few years. Nevertheless, the production of heavy quarkonium is still puzzling us after almost forty years since the discovery of J/ψ . New data have been taken at e^+e^- , ep and $p\bar{p}$ colliders, and a wealth of fixed-target data also exist and with the advances in theory and tremendous amount of precise data from the Large Hadron Collider (LHC), it is an excellent time to study the physics of heavy quarkonium production.

Nowadays, it is well-known that the dominant mechanism to produce the heavy quarkonia at high transverse momentum is fragmentation; the production of a parton with a large transverse momentum which subsequently decays to form a jet containing the expected hadron [2]. It is hence important to obtain the corresponding fragmentation function (FF), in order to properly estimate the production rate of a specific quarkonium state. Because of the simple internal structure of heavy quarkonia the perturbative QCD approximations to their FFs are well-defined in the nonrelativistic QCD (NRQCD) factorization framework [3]. To calculate the FFs, beside the current phenomenological approaches which are based on the χ^2 analysis of experimental data (see our previous work [4]), there are two theoretical schemes which are based on the fact that the FFs for hadrons containing heavy quarks can be computed analytically using the

perturbative QCD (pQCD) [5, 6]. In these theoretical schemes the QCD improved parton model provides a great theoretical frame to extract the FFs.

The first scheme for the inclusive production of heavy quarkonium has been developed by Bodwin, Braaten, and Lepage, who proposed a method on the basis of pQCD and the nonrelativistic quark model [3, 5–7]. In this model, the FF of a heavy quark Q into the pseudoscalar or vector heavy-light mesons $Q\bar{q}$ is defined as the cross section for producing a $Q\bar{q}$ -meson plus a light quark q with total four-momentum K^μ , divided by the cross section for producing an on-shell Q with the same three-momentum \mathbf{K} , while $K_0 \rightarrow \infty$. The fragmentation function in the Braaten's model is defined as [7]

$$D(z, \mu_0) = \frac{1}{16\pi^2} \int ds \lim_{K_0 \rightarrow \infty} \frac{\sum |M|^2}{\sum |M_0|^2}, \quad (1)$$

where $s = K^2$ is the invariant mass of the meson, M is the matrix element for producing mesons plus a light quark q , and M_0 is the matrix element for producing an on-shell Q . Here, D stands for the fragmentation function and z is the fragmentation parameter which refers to the longitudinal momentum fraction of the quarkonium and μ_0 is a fragmentation scale. This model was applied to the fragmentation processes $\bar{b} \rightarrow B_c$ and $\bar{b} \rightarrow B_c^*$ in Ref. [8]. Another elaborate model is proposed by Suzuki [9, 10] which is based on the convenient Feynman diagrams and the wave function of the respective heavy meson, where the wave function includes the effect of long-distance in the fragmentation. In this model, the heavy FFs are calculated using a diagram similar to that in Fig. 3, so the analytical expression of FFs depends on the transverse momentum p_T of the parton which appears as a phenomenological parameter (e.g. see Eq. (20)), while in the Braaten's model the integrations over all freedom degrees are performed.

Note that, as soon as more than one hadron is appeared

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in a hard scattering process, it is necessary to take into account the transverse momentum p_T of the partons. For example, transverse momentum dependent FFs show up explicitly in several semi-inclusive cross sections, in particular in azimuthal asymmetries. In the calculation of QCD corrections to these cross sections, the inclusion of the p_T dependent FFs will be essential. Experimentally, based on the 1992-1993 run (run 1A), the CDF collaboration published data on their first measurement of the B-meson differential cross section $d\sigma/dp_T$ for the exclusive decays $B^+ \rightarrow J/\psi K^+$ and $B^0 \rightarrow J/\psi K^{*0}$ [11]. In [12], the p_T distribution $d\sigma/dp_T$ is considered and the theoretical predictions are compared with the CDF data [11] for which $5\text{GeV} < p_T < 20\text{GeV}$. This is our main motivation to study the transverse momentum dependent FFs in the Suzuki's model. In Ref. [13], using this model we computed an exact analytical expression of the fragmentation function for c-quark to split into the D^+/D^0 mesons to LO. There, we investigated that there is an excellent consistency between our result and the current well-known phenomenological models and also with the experimental data from BELLE and CLEO.

In high energy processes, the main contribution of charmonium production results from gluon fragmentation, while the charm quark fragmentation contribution is much too small [14, 15]. This point was confirmed by the comparison between the theoretical predictions and the CDF J/ψ production data. Therefore, in this work using the Suzuki's model we focus on the gluon fragmentation into a vector (J/ψ) and pseudoscalar (η_c) S-wave charmonium to leading order of perturbative QCD and we present, for the first time, an analytical expression for the transverse momentum dependent fragmentation functions of $g \rightarrow \eta_c, J/\psi$.

Note that, in the past few years the $g \rightarrow \eta_c, J/\psi$ FFs have been calculated numerically, using the Braaten's model [2, 16]. In these papers, due to the lengthy and cumbersome expression of the $g \rightarrow J/\psi$ FF the two-dimensional integrals have been presented that must be evaluated numerically. Moreover, since in the Braaten's model integrations over all freedom degrees are performed, then the presented FFs are independent of the transverse momentum of the initial gluon.

Our analytical expression for the $g \rightarrow J/\psi$ FF will be compared with the numerical result presented in [16]. Using the FF obtained, we also calculate the first two moments of FF which are of phenomenological interest and subject to experimental determination. They correspond to the $g \rightarrow J/\psi$ branching fraction and the average energy fraction of the J/ψ meson which receives from the gluon.

This paper is organized as follows. In Sec. II, we explain our theoretical approach to calculate the FFs using the pQCD. Our analytical results of the $g \rightarrow \eta_c, J/\psi$ FFs will be presented in the Suzuki's model. In Sec. III, our numerical results for the gluon FFs are presented. Our conclusion is summarized in Sec. IV.

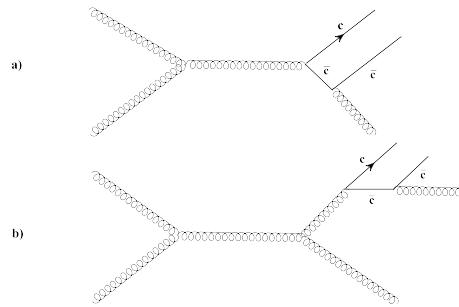


Figure 1: Feynman diagrams that contribute to charmonium production: (a) $gg \rightarrow c\bar{c}g$ at order- α_s^3 , (b) $gg \rightarrow c\bar{c}gg$ at order- α_s^4 [2].

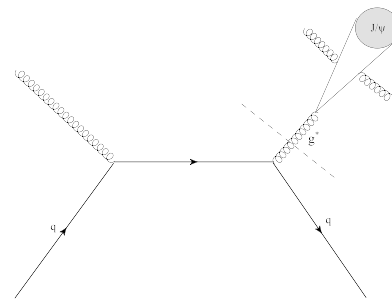


Figure 2: Feynman diagram of process $qq \rightarrow qq^* \rightarrow J/\psi gg$ [16].

II. GLUON FRAGMENTATION INTO S-WAVE CHARMONIUM: PERTURBATIVE QCD SCHEME

The theoretical schemes for calculating the charmonium FFs are based on the fact that the FFs for hadrons containing a heavy quark can be computed theoretically using perturbative QCD [2, 5, 6, 17]. The $g \rightarrow \eta_c, J/\psi$ FFs have been already calculated in Refs. [2, 16], using the Braaten's model where the fragmentation function is defined as in (1). In Ref. [2], authors considered the typical Feynman diagrams contributed to the production of charmonium states at the order α_s^3 (Fig. 1a) and at the order α_s^4 (Fig. 1b). They have pointed out that in most regions of phase space, the virtual gluons in Fig. 1b are off their mass shells by amounts of order p_T , where p_T refers to the large transverse momentum of charmonium state, and the contribution from this diagram is suppressed relative to the diagram in Fig. 1a by a power of $\alpha_s(p_T)$ when the spin-singlet S-wave charmonium $\eta_c(^1S_0)$ is considered. Then, they have obtained the $g \rightarrow \eta_c$ FF at $\alpha_s(\mu = 2m_c)^2$ -order. Authors have also calculated the FF for a gluon into J/ψ considering the Feynman diagram for the process $g^* \rightarrow J/\psi gg$ (Fig. 1b) to α_s^3 . They have not presented an analytical form for the $D_g^{J/\psi}(z, \mu)$ and, instead, the result is given in a two-dimensional integral form that must be evaluated numerically. In [16],

using the Braaten's model authors have also obtained an integral form for the polarized and unpolarized initial FFs of gluon into the J/ψ at $\alpha_s(\mu = 2m_c)^3$, considering a specific physical process shown in Fig. 2.

Here, we focus on the gluon fragmentation into a point-like $c\bar{c}$ pair in the $^1S_0(\eta_c)$ and $^3S_1(J/\psi)$ states and derive, for the first time, an analytical form of their transverse momentum dependent FFs, using the Suzuki's model. The leading contribution to the short-distance scattering amplitude of the fragmentation process $g \rightarrow J/\psi$ arises from the partonic process $g \rightarrow c\bar{c}gg$ (Fig. 3) and is of order α_s^3 , as its dominant decay process is $J/\psi \rightarrow 3g$. The hard scattering amplitude of the fragmentation $g \rightarrow \eta_c$ results from the process $g \rightarrow c\bar{c}g$ (Fig. 4) and is of order α_s^2 .

The fragmentation parameter z is normally defined in the Lorentz boost invariant form as $z = (E^H + p_L^H)/(E^g + p_L^g)$, which is known as the light-cone form. This definition is hard to be employed in the application of the gluon FF, because it involves the transverse momentum of the resulting heavy quarkonium. Instead, the following non-covariant definition is usually used approximately

$$z = \frac{E^H}{E^g}. \quad (2)$$

When the fragmenting gluon momentum $|\vec{k}| \rightarrow \infty$, the definition (2) is equivalent to the light-cone definition [16]. Therefore, we adopt the infinite momentum frame where the fragmentation parameter is defined as in (2). In Ref. [16], authors analysed the uncertainties induced by different definitions of the momentum fraction z , including the covariant and non-covariant definitions and showed that the FFs corresponding to the light-cone definition of the fragmentation parameter z are equivalent to the ones in the infinite momentum frame of gluon. Instead, the non-covariant definition (2) is used as an approximation unless $|\vec{k}| \rightarrow \infty$.

With the large heavy quark mass, the relative motion of the heavy quark pair inside the charmonium is effectively nonrelativistic [18]. For example, the squared relative velocity of the heavy quark pair in the quarkonium rest frame is $v^2 \approx 0.22$ for the J/ψ and $v^2 \approx 0.1$ for the Υ [19]. The nonrelativistic assumption allows us to use a simple mesonic wave function which is the solution of the Schrodinger equation with a Coulomb potential [13]. A typical simple mesonic wave function is given in [20] which is the nonrelativistic limit of the Bethe-Salpeter equation with the QCD kernel. Here, according to the Lepage-Brodsky's approach [21] we also neglect the relative motion of the heavy quark pair inside the charmonium states and we assume, for simplicity, that the quark pair are emitted collinearly with each other and move along the Z -axes. This assumption allows us to estimate the nonrelativistic mesonic wave function as a delta function form (more detail can be found in [13]).

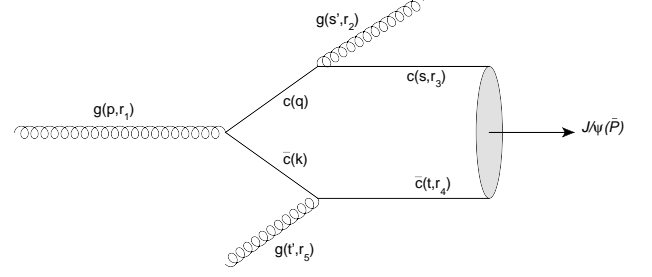


Figure 3: Feynman diagram of process $g \rightarrow J/\psi gg$.

A. J/ψ In gluon fragmentation

To derive an analytical result of the function $D_g^{J/\psi}(z, \mu)$ which refers to the fragmentation of a gluon into the J/ψ , we consider the Feynman diagram shown in Fig. 3, which is the subprocess of the physical process presented in Fig. 2. The spins (r_i) and the four-momenta of meson and partons are also labelled in Fig. 3. According to our early assumption, the meson and its constituent quarks move along the Z -axes (fragmentation axes). Then we set the relevant four-momenta as

$$\begin{aligned} s_\mu &= [s_0, \mathbf{0}, s_L] & t_\mu &= [t_0, \mathbf{0}, t_L] \\ s'_\mu &= [s'_0, \mathbf{s}'_\mathbf{T}, s'_L] & t'_\mu &= [t'_0, \mathbf{t}'_\mathbf{T}, t'_L] \\ p_\mu &= [p_0, \mathbf{p}_\mathbf{T}, p_L] & \bar{P}_\mu &= [\bar{P}_0, \mathbf{0}, \bar{P}_L]. \end{aligned} \quad (3)$$

Considering the definition of fragmentation parameter, $z = E^H/E^g = \bar{P}_0/p_0$ (2), we also may write the parton energies in terms of the initial gluon energy p_0 as

$$\begin{aligned} s_0 &= x_1 z p_0 & t_0 &= x_2 z p_0 \\ s'_0 &= x_3 (1 - z) p_0 & t'_0 &= x_4 (1 - z) p_0, \end{aligned} \quad (4)$$

where $x_1 = s_0/\bar{P}_0$ and $x_2 = t_0/\bar{P}_0$ are the meson energy fractions carried by the constituent quarks which the condition $x_1 + x_2 = 1$ holds for them. Following Refs. [22, 23], in the nonrelativistic approximation we assume that the contribution of each constituent quark from the meson energy is proportional to its mass, i.e. $x_1 = m_c/M$ and $x_2 = m_{\bar{c}}/M$ where $M = m_{J/\psi}$. Furthermore, we assume that the two gluon jets move almost in the same direction. This assumption is justified due to the fact that the very high momentum of the initial gluon is predominantly carried in the forward direction. Due to momentum conservation, the total transverse momentum of the two gluon jets will be identical to the transverse momentum of the initial gluon. Therefore, we have $\mathbf{t}'_\mathbf{T} = \mathbf{s}'_\mathbf{T} = \mathbf{p}_\mathbf{T}/2$. By this assumption one also has $x_3 = x_4 = 1/2$ in (4).

In the Suzuki's model the fragmentation function $D_g^{J/\psi}(z, \mu)$ at the initial scale $\mu_0 = m_{J/\psi}$, is obtained by squaring the total amplitude and integrating over fi-

nal state phase space [10, 23]

$$D_g^{J/\psi}(z, \mu_0) = \frac{1}{1+2r_1} \sum_s \int |T_M|^2 \times \delta^3(\bar{\mathbf{P}} + \mathbf{s}' + \mathbf{t}' - \mathbf{p}) d^3\bar{\mathbf{P}} d^3\mathbf{s}' d^3\mathbf{t}', \quad (5)$$

where, r_1 is the spin of initial gluon and T_M is the probability amplitude of the J/ψ production. The amplitude T_M involves the hard scattering amplitude T_H , which can be computed perturbatively from partonic subprocesses, and the process-independent distribution amplitude Φ_M which contains the bound state nonperturbative dynamic of outgoing meson, i.e.

$$T_M(\bar{\mathbf{P}}, p, s', t') = \int [dx_i] T_H(\bar{\mathbf{P}}, p, s', t', x_i) \Phi_M(x_i, Q^2), \quad (6)$$

where, $[dx_i] = dx_1 dx_2 \delta(1 - x_1 - x_2)$ and x_i 's are the momentum fractions carried by the constituent quarks. This scheme, introduced in [20, 24], is applied to absorb the soft behaviour of the bound state into the scattering amplitude T_H . The amplitude T_H is, in essence, the partonic cross section to produce a heavy quark pair $c\bar{c}$ with certain quantum numbers that, in the old fashioned perturbation theory is expressed as

$$T_H = \frac{(4\pi\alpha_s(2m_c))^{\frac{3}{2}} m_c^2 C_F}{2\sqrt{2}\bar{P}_0 p_0 s'_0 t'_0} \frac{\Gamma}{(\bar{P}_0 + t'_0 + s'_0 - p_0)}, \quad (7)$$

where α_s is the strong coupling constant, $C_F = \sqrt{5}/12$ is the color factor and Γ represents an appropriate combination of the quark propagators and the spinorial parts of the amplitude. It reads

$$\Gamma = G_1 G_2 \left\{ \bar{u}(s, r_3) \epsilon_2^*(q + m_c) \epsilon_1(\not{k} + m_c) \epsilon_3^* v(t, r_4) \right\}. \quad (8)$$

Here, ϵ_i 's are the gluon polarization vectors, $G_1 = 1/(q^2 - m_c^2) = 1/(2s.s')$ and $G_2 = 1/(k^2 - m_c^2) = 1/(2t.t')$ are proportional to the quark propagators. We put the dot products of the relevant four-vectors in the following form

$$t.t' = s.s' = \frac{zm_c}{4M(1-z)} p_T^2 + \frac{M(1-z)m_c}{4z}. \quad (9)$$

In (6), Φ_M is the process-independent probability amplitude to find quarks co-linear up to a scale Q^2 in the mesonic bound state, so that by working in the infinite-momentum frame it can be estimated as a delta function. Therefore, the distribution amplitude for a S-wave heavy meson with neglecting the Fermi motion, reads [20, 25]

$$\Phi_M \approx \frac{f_M}{2\sqrt{3}} \delta(x_1 - \frac{m_c}{M}), \quad (10)$$

where M is the meson mass and $f_M = \sqrt{12/M} |\Psi(0)|$ is the meson decay constant which is related to the non-relativistic mesonic S-wave function $\Psi(0)$ at the origin.

Substituting Eqs. (6), (7) and (10) in (5) and carrying out the necessary integrations, the fragmentation function reads

$$D_g^{J/\psi}(z, \mu_0) = \frac{2}{3} (\pi\alpha_s)^3 (f_M C_F m_c^2)^2 \times \int \frac{\frac{1}{3} \sum_s \bar{\Gamma} \Gamma \delta^3(\bar{\mathbf{P}} + \mathbf{s}' + \mathbf{t}' - \mathbf{p})}{(\bar{P}_0 p_0 s'_0 t'_0) (\bar{P}_0 + s'_0 + t'_0 - p_0)^2} d^3\bar{\mathbf{P}} d^3\mathbf{s}' d^3\mathbf{t}'. \quad (11)$$

To obtain an analytical form of the fragmentation function for gluon to split into the S-wave charmonia (i.e. $\eta_c, J/\psi$), we apply the scenario introduced in [26]. According to this scenario if $v(t)$ and $\bar{u}(s)$ are the Dirac spinors of the quarks forming the charmonium bound states, in the nonrelativistic approximation the projection operator is defined as

$$\Lambda_{S, S_z}(s, t) = v(t) \bar{u}(s) \propto (t + m_c) \Pi_{S, S_z}, \quad (12)$$

where Π_{S, S_z} is the appropriate spin projection operator; $\Pi_{00} = \gamma_5$ for pseudoscalar state (η_c) and $\Pi_{1S_z} = \not{\epsilon}(S_z)$ for vector state (J/ψ). The spin content of the polarized meson is then given by either γ_5 or $\not{\epsilon}$, as could well be expected. This operator is convenient for our assumption in which we ignore the Fermi motion, so that the constituent quarks will fly together in parallel. Therefore, the spinorial part of the amplitude for formation of the vector charmonium state J/ψ , which we denote by V , may be presented in the following form

$$\Gamma^V \propto G_1 G_2 \left\{ (t + m_c) \not{\epsilon} \epsilon_2^*(q + m_c) \not{\epsilon}_1(\not{k} + m_c) \epsilon_3^* \right\}, \quad (13)$$

where $q = s + s'$ and $k = t + t'$ are the energy-momenta of the virtual intermediate quarks, ϵ is the polarization four-vector of the meson J/ψ which may be in a longitudinal state $\epsilon^{(L)\mu} = \epsilon^\mu(\bar{P}, \lambda = 0)$ or a transverse state $\epsilon^{(T)\mu} = \epsilon^\mu(\bar{P}, \lambda = \pm 1)$. These components satisfy the relations; $\epsilon(\bar{P}, \lambda) \cdot \bar{P} = 0$, $\epsilon^{(T)} \cdot \bar{\mathbf{P}} = 0 = \epsilon^{(L)} \times \bar{\mathbf{P}}$ and $\epsilon(\bar{P}, \lambda) \cdot \epsilon^*(\bar{P}, \lambda') = -\delta_{\lambda, \lambda'}$. Therefore, for a vector charmonium with the four-momentum $\bar{P}^\mu = [\bar{P}_0; \mathbf{0}, \bar{P}_L]$, the polarization four-vector is expressed as

$$\begin{aligned} \epsilon^{(L)\mu} &= \frac{1}{M} (\bar{P}_L; 0, 0, \bar{P}_0), \\ \epsilon^{(T)\mu} &= \frac{1}{\sqrt{2}} (0; \mp 1, -i, 0), \end{aligned} \quad (14)$$

where $\bar{P}_L = s_L + t_L$ and $\bar{P}_0 = s_0 + t_0$. Now to obtain an analytical form of the J/ψ FF, in (11) we perform a sum over the colors and the spins of gluons. Then the amplitude squared $\bar{\Gamma} \Gamma$ reads

$$\begin{aligned} \sum_s \Gamma^V \bar{\Gamma}^V &= 4G_1^2 G_2^2 Tr[(t + m_c) \not{\epsilon} \gamma^\mu (q + m_c) \gamma^\nu (\not{k} + m_c) \\ &\quad \times (\not{k} + m_c) \gamma_\nu (q + m_c) \gamma_\mu \not{\epsilon}^* (t + m_c)]. \end{aligned} \quad (15)$$

Using the traditional trace technique, the trace may be expressed as the dot products of four-vectors. Here, we

put the dot products of the relevant four-vectors in the following forms:

$$\begin{aligned}
t'.s' &= 0, \\
p.\epsilon_T &= -\frac{p_T}{\sqrt{2}}(-1+i), \\
p.\epsilon_L &= -\frac{M}{2z} + \frac{z}{2M}p_T^2, \\
p.s &= p.t = \frac{m_c^2}{z} + \frac{z}{4}p_T^2, \\
p.t' &= p.s' = \frac{z^2}{4(1-z)}p_T^2, \\
s'.\epsilon_T &= \epsilon_T.t' = -\frac{p_T}{2\sqrt{2}}(-1-i), \\
t.s' &= s.t' = \frac{1-z}{2z}m_c^2 + \frac{z}{8(1-z)}p_T^2, \\
s'.\epsilon_L &= \epsilon_L.t' = -\frac{M(1-z)}{4z} + \frac{z}{4M(1-z)}p_T^2. \quad (16)
\end{aligned}$$

To proceed we need to specify the phase space integrations in (11). Note that

$$\int \frac{d^3\bar{\mathbf{P}}\delta^3(\bar{\mathbf{P}} + \mathbf{t}' + \mathbf{s}' - \mathbf{p})}{\bar{P}_0(\bar{P}_0 + s'_0 + t'_0 - p_0)^2} = \frac{\bar{P}_0}{(M^2 + 2p.t' + 2p.s' - 2s'.t')^2}. \quad (17)$$

Here, instead of performing the transverse momentum integrations we replace the integration variable by its average value $\langle p_T^2 \rangle$ in each case, which is a free parameter and can be specified experimentally. Therefore we can write

$$\begin{aligned}
\int F(z, \mathbf{s}'_T) d^3s' &= \int F(z, \mathbf{s}'_T) ds'_L d^2s'_T \\
&\approx m_c^2 s'_0 F(z, \langle s'^2_T \rangle) = m_c^2 s'_0 F(z, \frac{1}{4} \langle p_T^2 \rangle), \quad (18)
\end{aligned}$$

and

$$\begin{aligned}
\int F'(z, \mathbf{t}'_T) d^3t' &= \int F'(z, \mathbf{t}'_T) dt'_L d^2t'_T \\
&\approx m_c^2 t'_0 F'(z, \langle t'^2_T \rangle) = m_c^2 t'_0 F'(z, \frac{1}{4} \langle p_T^2 \rangle). \quad (19)
\end{aligned}$$

Finally, putting all in (11) and by assuming $M = 2m_c$ in the nonrelativistic limit, we obtain the longitudinal and the transverse components of the $g \rightarrow J/\psi$ FF, as

$$\begin{aligned}
D_{g \rightarrow J/\psi}^T(z, \mu_0) &= \frac{8N_1 z \alpha_s^3}{g(z, \langle p_T^2 \rangle)} \left[-16 \frac{1-z^2}{z^2} m_c^6 \right. \\
&\quad \left. - \frac{8}{z(1-z)} (2z^3 + 4z^2 + 3z - 4) m_c^4 p_T^2 \right. \\
&\quad \left. + \frac{1}{(1-z)^3} \left(z(17z^2 - 25z + 8) m_c^2 p_T^4 - z^4 p_T^6 \right) \right], \quad (20)
\end{aligned}$$

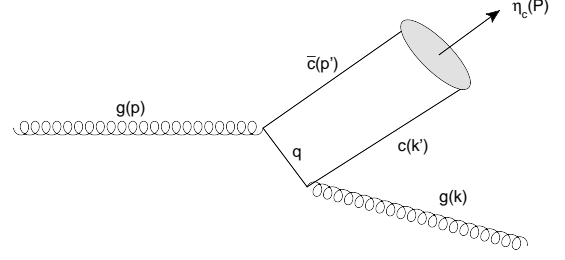


Figure 4: Feynman diagram of process $g \rightarrow \eta_c$ at α_s^2 -order.

and

$$\begin{aligned}
D_{g \rightarrow J/\psi}^L(z, \mu_0) &= \frac{4N_1 z \alpha_s^3}{g(z, \langle p_T^2 \rangle)} \left[-32 \frac{1-z^2}{z^2} m_c^6 \right. \\
&\quad \left. + \frac{16}{1-z} (3z^2 + 2z + 2) m_c^4 p_T^2 + \right. \\
&\quad \left. \frac{1}{(1-z)^3} \left(2z^2(-4z^2 - 7z + 11) m_c^2 p_T^4 + 3z^4 p_T^6 \right) \right]. \quad (21)
\end{aligned}$$

where $N_1 = (\pi^3 m_c^8 f_M^4 C_F^2)/216$, and

$$\begin{aligned}
g(z, \langle p_T^2 \rangle) &= \left[\frac{z m_c}{2M(1-z)} p_T^2 + \frac{M(1-z) m_c}{2z} \right]^4 \\
&\quad \times \left[M^2 + \frac{z^2}{1-z} p_T^2 \right]^2. \quad (22)
\end{aligned}$$

Note that, the fragmentation function for a vector charmonium J/ψ is the sum of the longitudinal and twice the transverse components, i.e. $D_{g \rightarrow J/\psi}^{J/\psi}(z, \mu_0) = 2D_g^T + D_g^L$. Finally, we point out that to impose the effect of quarkonium spin into the calculation, a second scenario is defined in [22, 27], where in the nonrelativistic approximation the spin projection operator is expressed as

$$\Lambda_{S,S_z}(\bar{P}) = \frac{f_M}{\sqrt{48}} (\bar{\not{P}} + M) \Pi_{SS_z}, \quad (23)$$

where, f_M is the meson decay constant and M and \bar{P} are the mass and the four-momentum of meson bound state, respectively. In the nonrelativistic limit ($\bar{P} = 2\not{t}/M = 2m_c$), these two scenarios (12, 23) are the same and, in conclusion, the sum of transverse and longitudinal polarisations of the J/ψ should be identical.

B. η_c In gluon fragmentation

Following Ref. [2], the fragmentation function for a gluon to split into the pseudoscalar S-wave charmonium η_c can be calculated by considering Fig. 1a. To derive

an analytical result of the $g \rightarrow \eta_c$ FF we consider the Feynman diagram shown in Fig. 4, where a gluon forms a bound state $c\bar{c}$ with a gluon produced through a single c -quark. The four-momenta of meson and partons are also labelled. According to our previous assumption the meson and its constituent quarks move along the Z -axes (fragmentation axes). Then one can set the relevant four-momenta as

$$\begin{aligned} p_\mu &= [p_0, \mathbf{p}_T, p_L] & k_\mu &= [k_0, \mathbf{p}_T, k_L] \\ p'_\mu &= [p'_0, \mathbf{0}, p'_L] & k'_\mu &= [k'_0, \mathbf{0}, k'_L] \\ \bar{P}_\mu &= [\bar{P}_0, \mathbf{0}, \bar{P}_L]. \end{aligned} \quad (24)$$

Considering the definition of fragmentation parameter (2), we may write the parton energies in terms of the gluon energy p_0 as; $p'_0 = x_1 z p_0$, $k'_0 = x_2 z p_0$, $k_0 = (1-z)p_0$ where x_1 and x_2 are the meson energy fractions carried by the constituent quarks. Following our early discussion, the contribution of each constituent quark from the meson energy is proportional to its mass, i.e. $x_1 = m_c/M$ and $x_2 = m_{\bar{c}}/M$ where $M = m_{J/\psi}$.

We start with the definition of the fragmentation function as

$$D_g^{\eta_c} = \frac{1}{1+2s_g} \sum_s \int |T_M|^2 \delta^3(\bar{\mathbf{P}} + \mathbf{k} - \mathbf{p}) d^3\bar{\mathbf{P}} d^3\mathbf{k}, \quad (25)$$

where the probability amplitude T_M is related to the hard scattering amplitude T_H and the distribution amplitude Φ_M as in (6). In view of our early discussion in this section, we propose a delta function as (10) for the amplitude Φ_M .

Using the perturbation theory, the amplitude T_H is written in the following form

$$T_H = \frac{4\pi\alpha_s(2m_c)m_c^2 C_F}{2\sqrt{2P_0 p_0 k_0}} \frac{\Gamma}{(\bar{P}_0 + k_0 - p_0)}, \quad (26)$$

where Γ represents the spinorial parts of the amplitude as

$$\Gamma = G \left\{ \bar{u}(k', r_2) \epsilon_2^*(k) (q' + m_c) \epsilon_1(p) v(p', r_1) \right\}. \quad (27)$$

Here, $G = 1/(q^2 - m_c^2) = 1/(2k \cdot k')$ is proportional to the quark propagator and $C_F = 1/(2\sqrt{3})$ is the color factor.

Putting all in (25), the fragmentation function reads

$$\begin{aligned} D_g^{\eta_c}(z, \mu_0) &= \frac{1}{6} (\pi\alpha_s f_M C_F m_c^2)^2 \\ &\times \int \frac{1}{2} \sum_s \bar{\Gamma} \Gamma d^3\mathbf{k} \int \frac{\delta^3(\bar{\mathbf{P}} + \mathbf{k} - \mathbf{p})}{P_0(\bar{P}_0 + k_0 - p_0)^2} d^3\bar{\mathbf{P}}. \end{aligned} \quad (28)$$

Considering the scenario introduced in previous section (12), where the spin projection operator for a pseudoscalar charmonium state η_c is $\Pi_{00} = \gamma_5$, the spinorial

part of the amplitude (27) is presented in the following form

$$\Gamma^P \propto G \left\{ (\not{k}' + m_c) \gamma_5 \epsilon_2^*(k) (\not{q}' + m_c) \epsilon_1(p) \right\}. \quad (29)$$

By performing a sum over the colors and the spins of gluons, the amplitude squared reads

$$\sum_s \Gamma^P \bar{\Gamma}^P = 128 G^2 m_c^2 \left\{ 2m_c^2 + 2k \cdot k' + p' \cdot (k + k') \right\}. \quad (30)$$

Next we consider the phase space integrations (28). Following our previous approach, we have

$$\int \frac{d^3\bar{\mathbf{P}} \delta^3(\bar{\mathbf{P}} + \mathbf{k} - \mathbf{p})}{P_0(\bar{P}_0 + k_0 - p_0)^2} = \frac{\bar{P}_0}{(M^2 + 2p \cdot k)^2}, \quad (31)$$

and

$$\int F(z, p_T^2) d^3\mathbf{k} = m_c^2 k_0 F(z, \langle p_T^2 \rangle). \quad (32)$$

In order to get the correct result for the fragmentation function of the process $g \rightarrow \eta_c$, it is necessary to consider a second diagram that can be obtained from Fig. 4 by interchanging the two vertices where the gluons attached to the heavy quark lines.

In conclusion, in the referred scenario the fragmentation function of the process $g \rightarrow \eta_c$ is expressed as

$$\begin{aligned} D_{g \rightarrow \eta_c}(z, \mu_0) &= \frac{N_2 \alpha_s^2 z}{g(z, \langle p_T^2 \rangle)} (192 m_c) \\ &\times \left[2m_c + \frac{z p_T^2}{M(1-z)} + \frac{M(1-z)}{z} \right]. \end{aligned} \quad (33)$$

where $N_2 = (\pi m_c^3 f_M^2 C_F)^2 / 864$, and

$$g(z, \langle p_T^2 \rangle) = \left[\left(M^2 + \frac{z^2 p_T^2}{1-z} \right) \left(\frac{z^2 p_T^2 + M^2(1-z)^2}{M z(1-z)} \right) \right]^2. \quad (34)$$

III. RESULTS AND DISCUSSION

We are now in a position to present our phenomenological predictions for the gluon fragmentation into the η_c and J/ψ , by performing a numerical analysis. In general, the fragmentation function $D_g^H(z, \mu)$ depends on the factorization scale μ . Here, we have set the scale in the FF and in the running coupling constant $\alpha_s(\mu)$ to $\mu = 2m_c$ which is the minimum value of the invariant mass of the fragmenting gluon. Therefore, the functions (20, 21) and (33) should be regarded as models for the $g \rightarrow \eta_c, J/\psi$ FFs at the initial scale $\mu_0 = 2m_c$. For values of μ much larger than μ_0 , the initial FFs should be evolved from the

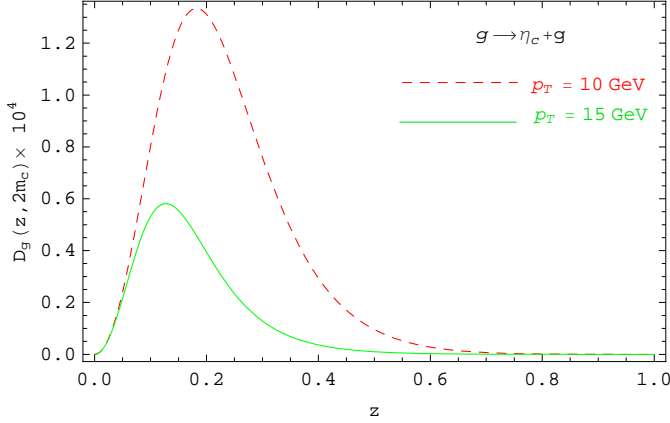


Figure 5: The fragmentation function for the process $g \rightarrow \eta_c$, considering two values of the transverse momentum $p_T = 10, 15$ GeV. The initial scale is set as $\mu_0 = 2m_c$.

scale μ_0 to the higher scale μ using the Altarelli-Parisi equations [28, 29]

$$\mu \frac{\partial}{\partial \mu} D_{i \rightarrow H}(z, \mu) = \sum_j \int_z^1 \frac{dy}{y} P_{i \rightarrow j}(z/y, \mu) D_{j \rightarrow H}(y, \mu), \quad (35)$$

where $P_{i \rightarrow j}(x, \mu)$ are the Altarelli-Parisi functions for the splitting of the parton of type i into a parton of type j with momentum fraction x . The only boundary condition on this evolution equation is the fragmentation function $D_{i \rightarrow H}(z, \mu_0)$ at the scale $\mu_0 = 2m_c$.

For numerical results, we take $m_c = 1.5$ GeV, $m_{J/\psi} = 3096.9$ MeV, $m_{\eta_c} = 2983.6$ MeV, $\alpha_s(2m_c) = 0.26$ and $f_M(c\bar{c}) = 0.48$ GeV [30].

In Fig. 5, our prediction for the $g \rightarrow \eta_c$ FF (33) by considering two values of the gluon transverse momentum ($p_T = 10, 15$ GeV) is shown. It shows that the fragmentation function distribution relies on the momentum p_T of the initial gluon. In Fig. 6, the behaviour of the $g \rightarrow J/\psi$ FF is studied. Here, $D_g^{J/\psi}$ is the convenient summation of the longitudinal and transverse fragmentation functions as $D_g^{J/\psi}(z, \mu_0) = 2D_g^T + D_g^L$, see Eqs. (20, 21). Our result shown in Fig. 6, is in acceptable agreement with the result presented in Fig. 3 of Ref. [16], when the non-covariant definition of fragmentation parameter (2) is applied. In both results, the peak position of the FF occurs at $z \approx 0.22$ when $p_T = 10$ GeV is considered, and the maximum value of the FF is $D_g^{J/\psi} \approx 0.8 \times 10^{-5}$. Besides the $g \rightarrow J/\psi$ FF itself, also its first two moments are of phenomenological interest. They correspond to the $g \rightarrow J/\psi$ branching fraction

$$B(\mu) = \int_0^1 dz D_{g \rightarrow J/\psi}(z, \mu), \quad (36)$$

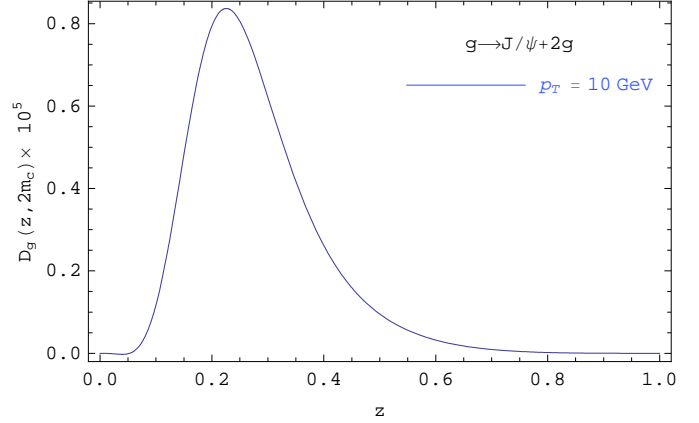


Figure 6: The fragmentation function for the process $g \rightarrow J/\psi$ at α_s^3 . Here, $D_g^{J/\psi}(z, \mu_0) = 2D_g^T + D_g^L$ where the transverse component D_g^T is given in (20) and the longitudinal one D_g^L is given in (21).

and the average energy fraction that the J/ψ meson receives from the gluon

$$\langle z \rangle(\mu) = \frac{1}{B(\mu)} \int_0^1 z dz D_{g \rightarrow J/\psi}(z, \mu). \quad (37)$$

Indeed, an order of magnitude estimate of the gluon fragmentation contribution to J/ψ production in any high transverse momentum process can be obtained by multiplying the cross section for producing gluons with $p_T > 2m_c$ by the branching fraction, which refers to the fragmentation probability. Our result for the $g \rightarrow J/\psi$ branching fraction is $B_{J/\psi}(2m_c) = 2.94 \times 10^{-6}$ and $\langle z \rangle_{J/\psi}(2m_c) = 0.277$ which this initial fragmentation probability can be compared with the result presented in [2] where $B(2m_c) = 3.2 \times 10^{-6}$.

Our results may be directly applied to the S-wave bottomonium states $\Upsilon(3S_1)$ and $\eta_b(1S_0)$, except that m_c is replaced by $m_b = 4.5$ GeV and the decay constant $f_M(b\bar{b}) = 0.33$ GeV [30] is the appropriate constant for the bottomonium states. Since the b-quark is heavier than the c-quark, we expect that the peaks of the fragmentation functions shift significantly toward higher values of z . Our results for the $g \rightarrow \eta_c, J/\psi$ FFs will be checked by the comparison between the theoretical predictions and the experimental measurements of the heavy meson cross sections at the LHC.

IV. CONCLUSION

Understanding hadronization, the process by which a parton evolves into a hadron, is complicated by its intrinsically nonperturbative nature. In hadron colliders, at sufficiently large transverse momentum of the heavy quarkonium production the direct production schemes

are normally suppressed while the fragmentation mechanism becomes dominant. The fragmentation refers to the process of a parton with high transverse momentum which subsequently decays into the expected hadron [2]. Beside the phenomenological approaches, it is well-known that the fragmentation function which describes this process can be calculated using perturbative QCD. In this paper we, for the first time, gave out an analytical form for the leading color-singlet contribution to the fragmentation function for a gluon to split into the vector and pseudoscalar S-wave charmonia ($J/\psi, \eta_c$) at the initial scale $\mu = 2m_c$. We used a different model in getting them from the Braaten's model applied in other literatures, see Refs. [2, 16]. Our results depend on the transverse momentum of the initial gluon and shows a sim-

ple analytical form whereas in the Braaten's model, the integrations over all freedom degrees are performed and due to the lengthy and cumbersome expression of the FFs the results are presented as the two-dimensional integrals that must be evaluated numerically. Since the transverse momentum dependent FFs show up explicitly in semi-inclusive cross sections, therefore in the QCD corrections the inclusion of these dependent FFs will be necessary. Our result for the $g \rightarrow J/\psi$ FF is in acceptable consistency with the numerical result presented in Ref. [16], when one uses the normal definition of the fragmentation parameter (2). We also found that the fragmentation probability of a high energy gluon splitting into J/ψ is about 10^{-6} .

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- [1] N. Brambilla, S. Eidelman, B. K. Heltsley, R. Vogt, G. T. Bodwin, E. Eichten, A. D. Frawley and A. B. Meyer *et al.*, Eur. Phys. J. C **71** (2011) 1534 [arXiv:1010.5827 [hep-ph]].
 - [2] E. Braaten and T. C. Yuan, Phys. Rev. Lett. **71** (1993) 1673.
 - [3] G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51** (1995) 1125 [Erratum-ibid. D **55** (1997) 5853] [hep-ph/9407339].
 - [4] M. Soleymaninia, A. N. Khorramian, S. M. Moosavi Nejad and F. Arbabifar, Phys. Rev. D **88**, no. 5, 054019 (2013) [Addendum-ibid. D **89**, no. 3, 039901 (2014)] [arXiv:1306.1612 [hep-ph]].
 - [5] E. Braaten, K. -m. Cheung and T. C. Yuan, Phys. Rev. D **48** (1993) 4230.
 - [6] C. -H. Chang and Y. -Q. Chen, Phys. Lett. B **284** (1992) 127.
 - [7] E. Braaten, K. m. Cheung, S. Fleming and T. C. Yuan, Phys. Rev. D **51** (1995) 4819 [hep-ph/9409316].
 - [8] E. Braaten, K. m. Cheung and T. C. Yuan, Phys. Rev. D **48** (1993) 5049 [hep-ph/9305206].
 - [9] M. Suzuki, Phys. Lett. B **71** (1977) 139.
 - [10] M. Suzuki, Phys. Rev. D **33** (1986) 676.
 - [11] F. Abe *et al.* [CDF Collaboration], Phys. Rev. Lett. **75** (1995) 1451 [hep-ex/9503013].
 - [12] J. Binnewies, B. A. Kniehl and G. Kramer, Phys. Rev. D **58** (1998) 034016 [hep-ph/9802231].
 - [13] S. M. M. Nejad and A. Armat, Eur. Phys. J. Plus **128** (2013) 121 [arXiv:1307.6351 [hep-ph]].
 - [14] D. P. Roy and K. Sridhar, Phys. Lett. B **339** (1994) 141 [hep-ph/9406386].
 - [15] A. F. Falk, M. E. Luke, M. J. Savage and M. B. Wise, Phys. Lett. B **312** (1993) 486 [hep-ph/9305260].
 - [16] W. Qi, C. F. Qiao and J. X. Wang, Phys. Rev. D **75** (2007) 074012 [hep-ph/0701264].
 - [17] J. P. Ma, Nucl. Phys. B **506** (1997) 329.
 - [18] Y. Q. Ma, J. W. Qiu and H. Zhang, Phys. Rev. D **89** (2014) 094029 [arXiv:1311.7078 [hep-ph]].
 - [19] G. T. Bodwin, U.-R. Kim and J. Lee, JHEP **1211** (2012) 020 [arXiv:1208.5301 [hep-ph]].
 - [20] S. J. Brodsky and C. -R. Ji, Phys. Rev. Lett. **55** (1985) 2257.
 - [21] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22** (1980) 2157.
 - [22] K. Kolodziej, A. Leike and R. Ruckl, Phys. Lett. B **348**, 219 (1995) [hep-ph/9412249].
 - [23] M. A. Gomshi Nobary, J. Phys. G **20**, 65 (1994).
 - [24] A. D. Adamov and G. R. Goldstein, Phys. Rev. D **56** (1997) 7381.
 - [25] F. Amiri, B. C. Harms and C. -R. Ji, Phys. Rev. D **32** (1985) 2982.
 - [26] B. Guberina, J. H. Kuhn, R. D. Peccei and R. Ruckl, Nucl. Phys. B **174** (1980) 317.
 - [27] J. H. Kuhn, J. Kaplan and E. G. O. Safiani, Nucl. Phys. B **157** (1979) 125.
 - [28] V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. **15**, 438 (1972) [Yad. Fiz. **15**, 781 (1972)].
 - [29] G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).
 - [30] M. A. Gomshi Nobary, J. Phys. G **27** (2001) 21.